Year 12 Mathematics Task 1 2013 (Term 4 2012)

Solutions and Marking Guidelines Question No. 1

Outcomes Addressed in this Question
applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and H5

Serie Outcome	s to solve problems  Solutions	Marking Guidelines
H5	(a) D	1 mark Correct answer
H5	(b) B	1 mark Correct answer
Н5	(c) (i) $d_{AB} = \sqrt{8^2 + 6^2}$ $= \sqrt{100}$ $= 10 \text{ units}$ $d_{OC} = \sqrt{4^2 + 3^2}$ $= \sqrt{25}$ $= 5 \text{ units}$	2 marks Correct solution showing both distances correctly 1 mark Only one of the two distances calculated correctly
Н5	(ii) $m_{AB} = \frac{6}{8}$ Equation of AB $= \frac{3}{4}$ $y - y_1 = m(x - x_1)$ $y - 0 = \frac{3}{4}(x + 4)$ $y = \frac{3}{4}x + 3$ $3x - 4y + 12 = 0$ as required	2 marks Correct solution 1 mark Substantial progress towards correct solution, including gradient of line AB
Н5	(iii) $m_{AB} = \frac{3}{4}$ Shown above $m_{OC} = \frac{\text{rise}}{\text{run}}$ $= \frac{3}{4}$ $\therefore AB  OC \text{ (gradients are equal)}$ Quadrilateral OCBA is a trapezium (1 pair of opposite sides are paralle	2 marks Correct solution showing lines are parallel AND correctly names quadrilateral. 1 mark Shows only one part of the correct solution ie. Parallel lines OR name of quad.
Н5	(iv) $d_{perp.} = \frac{ Ax_1 + By_1 + C }{\sqrt{A^2 + B^2}}$ $= \frac{ 3 \times 4 - 4 \times 3 + 12 }{\sqrt{3^2 + 4^2}}$ $= \frac{12}{5}$	2 marks Correct solution 1 mark Incorrect answer but correct substitution into correct formula
Н5	(v)  Area of Trapezium $A = \frac{1}{2}h(a+b)$ $= \frac{1}{2} \times \frac{12}{5}(10+5)$ $= \frac{12}{10} \times 15$ $= 18 \text{ units}^2$	2 marks Correct solution OR correct solution obtained from incorrect values from earlier parts of the question 1 mark Incorrect answer but correct substitution into correct formula

Year 11 Question No. 2 Question soutine page routine page	Year 11  Year 11  Outstion No. 2  Solutions and Marking Guidelines  Outcomes Addressed in this Question  P3 performs routine arithmetic and algebraic manipulation  P4 changes and applies appropriate arithmetic, algebraic, graphical and geometric techniques	trace reasonation was a
Outcome	Solutions	Marking Guidelines
(a)	A	1 mark - correct answer
(b)	₽	1 mark - correct answer
<u>©</u>	(i) $2u^2 - 17u + 8 = 0$ (2u - 1)(u - 8) = 0	1 mark correct solution
	$u = \frac{1}{2}$ , 8	
	(ii) $2 \times 2^{2x} - 17(2^x) + 8 = 0$	2 marks Correct solution
	$2u^2 - 17u + 8 = 0$ where $u = 2^x$	I mark Partially correct solution
	$u = 2^x = \frac{1}{2}, 8$ x = -1, 3	Partially correct solution
(a)	$m^2 - 4m - 12 < 0$ (m-6)(m+2)<0	2 marks Correct solution
	-2 < m < 6	1 marks Partially correct solution
(c)	$\Delta = b^2 - 4ac$	
	$= (-k)^2 - 4(3)(3)$ = $k^2 - 36 > 0$ for real roots	2 marks Correct solution Imark
•	ic $k \le -6$ or $k \ge 6$	Partially correct solution
(5)	(i) $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$	1 mark – correct answer
	and $\alpha\beta = \frac{c}{a} = 2$	
	(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	
	$=\left(\frac{3}{2}\right)^2-2(2)$	I mark - correct answer
	" 1 4 7	
MANUAL TO THE STATE OF THE STAT	(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \beta}$	
	. ⊪ 3 2 ÷2 2	1 mark - correct answer
	11 W   4	

					Outcome	Question No. 3 Outcomes Addr
$ PM  = 2 \times  PN $ $ PM ^2 = 4 PN ^2$ $(x-2)^2 + (y-1)^2 = 4\{(x-5)^2 + (y-1)^2\}$ $x^2 - 4x + 4 + y^2 - 2y + 1 = 4x^4 - 40x + 100 + 4y^2 - 8y + 4$ $0 = 3x^2 - 36x + 3y^2 - 6y + 99$ $0 = x^2 - 12x + y^2 - 2y + 33$ $0 = (x-6)^2 + (y-1)^2 - 4$ $(x-6)^2 + (y-1)^2 = 4$ Circle, centre (6,1) and radius 2	(d) 3-2 3 4 5 6 7	(c) $x^2-4x+12=8y$ becomes $(x-2)^2+8=8y$ and $(x-2)^2=8(y-1)$ (i) vertex = (2,1) (ii) focal length = 2 (iii) focus = (2,3) (iv) directrix is $y = -1$	(b) B: $3x - y = 0$ (c) $x^2 - 4x + 12 = x^2 - 4x + 4 + 8$ $= (x - 2)^2 + 8$	(a) C: $(x-2)^2 = 8(y+3)$	Solutions	Solutions and Marking Guidelines essed in this Question: H2 P4 H5
1 mark  1 mark  1 mark  1 mark (for any version of the equation)	1 mark	1 mark 1 mark 1 mark 1 mark	1 mark for first lin3 2 marks for solution	1 mark for B	Marking Guidelines	AND TABLE TO A LACT TO A L

## HURLSTONE AGRICULTURAL HIGH SCHOOL



#### **MATHEMATICS**

# 2013 HSC

#### ASSESSMENT TASK 1

### Examiners ~ S Gee, G Rawson, B Morrison, S Faulds

#### **GENERAL INSTRUCTIONS**

- Reading Time 3 minutes.
- Working Time 40 MINUTES.
- Attempt all questions.
- All necessary working should be shown in every question.
- This paper contains three (3) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and Mathematical templates may be used.
- Each question is to be started on a new piece of paper.
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME:	
TEACHER	

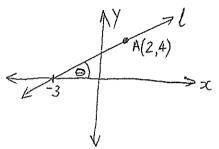
Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

- (a) The line perpendicular to 4x 3y + 5 = 0 is:
  - A: 4x 3y + 8 = 0

B: 4x + 3y - 7 = 0

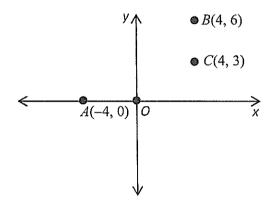
C: 3x - 4y = 0

- D: 3x + 4y + 1 = 0
- (b) The diagram below shows a student's sketch of a straight line, l.



What is the size of the angle  $\theta$  to the nearest degree? (The diagram is not to scale.)

- A: 16°
- B: 39°
- C: 74°
- D: 143°
- (c) The number plane below shows the points A(-4, 0), B(4, 6), C(4, 3) and the origin O.



(i) Calculate the length of the intervals AB and OC.

2

(ii) Show that the line AB has equation 3x - 4y + 12 = 0.

2

(iii) Show that AB||OC. What type of quadrilateral is OCBA?

2

(iv) Find the perpendicular distance of C from the line AB.

2

(v) Calculate the area of the quadrilateral *OABC*.

2

Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

- (a) The quadratic equation  $x^2 + 3x 1 = 0$  has roots  $\alpha$  and  $\beta$ . What is the value of  $\alpha\beta + (\alpha + \beta)$ ?
  - A: -4
- B: -2
- C: 2
- D: 4
- (b) What is the value of k if the equation  $2x^2 + 3x + k = 0$  has -3 as one of its roots?
  - A: –27
- B: -9
- C:
- D: 27

(c) (i) Solve the equation  $2u^2 - 17u + 8 = 0$ 

1

(ii) Hence or otherwise, solve  $2 \times 2^{2x} - 17(2^x) + 8 = 0$ 

2

(d) Find the values of m for which  $m^2 - 4m - 12 < 0$ .

2

(e) For what values of k does  $3x^2 - kx + 3 = 0$  have real roots?

2

- (f) For the quadratic equation  $2x^2 3x + 4 = 0$  with roots  $\alpha$  and  $\beta$ . Find the value of:
  - (i)  $\alpha + \beta$  and  $\alpha\beta$

1

(ii)  $\alpha^2 + \beta^2$ 

1

(iii)  $\frac{1}{\alpha} + \frac{1}{\beta}$ 

1

Clearly label and place the correct response for each multiple choice question at the start of your answer booklet.

(a) Which of the following parabolas has x = 2 as its axis of symmetry?

A:  $(x+2)^2 = 8(y-3)$ 

B:  $(v+2)^2 = 8(x-3)$ 

C:  $(x-2)^2 = 8(y+3)$ 

D:  $(y-3)^2 = 8(x-2)$ 

(b) The locus of a point that moves so that it is always equidistant from the points (-2,4) and (4,2)is given by the equation:

A:

3y-x=0 B: 3x-y=0 C: 3x-y+2=0 D: 3y-x+2=0

Show that  $x^2 - 4x + 12 = (x-2)^2 + 8$ (c)

2

Hence or otherwise, by expressing the parabola  $x^2 - 4x + 12 = 8y$  in the form  $(x-h)^2 = 4a(y-k)$ 

Find

(i) The vertex 1

(ii) The focal length 1

(iii) The focus 1

The equation of the directrix (iv)

1

- It is given that the point P(x, y) moves so that it is always twice the distance from a point (d) M(2,1) as it is from the point N(5,1).
  - Draw a clear diagram marking in all of the information. (i)

1

Find the locus of the point P. (ii)

3